

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

October/November 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



BLANK PAGE

- 1** Given that $\ln(1 + e^{2y}) = x$, express y in terms of x . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2 Solve the inequality $|2x - 3| > 4|x + 1|$.

[4]

[illegible]

- 3 The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\frac{dy}{dx} = \operatorname{cosec} 2t$.

[5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 4 The number of insects in a population t weeks after the start of observations is denoted by N . The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{dN}{dt} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{dN}{dt} = -0.01e^{-0.02t}N. \quad [1]$$

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) State what happens to the value of N as t becomes large. [1]

.....

.....

.....

.....

.....

.....

5 The curve with equation $y = e^{-2x} \ln(x - 1)$ has a stationary point when $x = p$.

(i) Show that p satisfies the equation $x = 1 + \exp\left(\frac{1}{2(x-1)}\right)$, where $\exp(x)$ denotes e^x . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Verify by calculation that p lies between 2.2 and 2.6.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Use an iterative formula based on the equation in part (i) to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 6 (i) By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x \operatorname{cosec}^2 x \, dx = \frac{1}{4}(\pi + \ln 4)$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 7** Two lines l and m have equations $\mathbf{r} = a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ respectively, where a is a constant. It is given that the lines intersect.

(i) Find the value of a .

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

8 Let $f(x) = \frac{x^2 + x + 6}{x^2(x + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

© UCLES 2019

9 (i) By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins or other markings on the paper.

(ii) Hence solve the equation $\cos 3x + 3 \cos x + 1 = 0$, for $0 \leq x \leq \pi$. [2]

.....

.....

.....

.....

.....

-
-
-
-
-
-
- (iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x \, dx$. [4]

- 10 (a)** The complex number u is given by $u = -3 - (2\sqrt{10})i$. Showing all necessary working and without using a calculator, find the square roots of u . Give your answers in the form $a + ib$, where the numbers a and b are real and exact. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - i| \leq 3$, $\arg z \geq \frac{1}{4}\pi$ and $\operatorname{Im} z \geq 2$, where $\operatorname{Im} z$ denotes the imaginary part of the complex number z . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.